Mon. Not. R. Astron. Soc. **380,** 1023–1028 (2007) doi:10.1111/j.1365-2966.2007.12126.x

Kinematic effect in gravitational lensing by clusters of galaxies

M. Sereno★
Institut für Theoretische Physik, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland

Accepted 2007 June 14. Received 2007 June 13; in original form 2007 March 16

ABSTRACT

Gravitational lensing provides an efficient tool for the investigation of matter structures, independent of the dynamical or the hydrostatic equilibrium properties of the deflecting system. However, it depends on the kinematic status. In fact, either a translational motion or a coherent rotation of the mass distribution can affect the lensing properties. Here, light deflection by galaxy clusters in motion is considered. Even if gravitational lensing mass measurements of galaxy clusters are regarded as very reliable estimates, the kinematic effect should be considered. A typical peculiar motion with respect to the Hubble flow brings about a systematic error $\lesssim 0.3$ per cent, independent of the mass of the cluster. On the other hand, the effect of the spin increases with the total mass. For cluster masses $\sim 10^{15}\,\mathrm{M}_{\odot}$, the effect of the gravitomagnetic term is $\lesssim 0.04$ per cent on strong lensing estimates and $\lesssim 0.5$ per cent in the weak-lensing analyses. The total kinematic effect on the mass estimate is then $\lesssim 1$ per cent, which is negligible in current statistical studies. In the weak-lensing regime, the rotation imprints a typical angular modulation in the tangential shear distortion. This would allow, in principle, a detection of the gravitomagnetic field and a direct measurement of the angular velocity of the cluster but the required background source densities are well beyond current technological capabilities.

Key words: gravitational lensing – galaxies: clusters: general – cosmology: observations.

1 INTRODUCTION

Clusters of galaxies are the biggest things whose masses can be reliably measured. The measurements of their properties are prerequisites to understand the structure in the Universe on a very large scale and to investigate processes associated with galaxy formation (Voit 2005). Investigations are often performed using rather strong assumptions. Mass estimates based on X-ray observations are routinely obtained through the hydrostatic equilibrium equation. Such measurements can be quite accurate if the temperature profile is well reconstructed from the projected measured one (Nagai, Vikhlinin & Kravtsov 2007) but they can be biased low by 5–10 per cent through the virial region primarily due to neglecting the unknown pressure support provided by gas bulk motion (Rasia et al. 2006; Nagai et al. 2007). The complex thermal structure of the emitting plasma can also bias towards lower values (Rasia et al. 2006). The mass of a steady cluster can also be inferred putting the observed velocity dispersion through the virial theorem (Voit 2005 and references therein). However, assumptions must be made on the degree of anisotropy to relate the projected velocity dispersion to the intrinsic components. A boundary pressure term can also alter the viral relation.

The hypotheses of either hydrostatic or dynamical equilibrium might not be suitable in many systems. Clusters of galaxies are the latest objects to form in a hierarchical cold dark matter (CDM) scenario and many of them are likely to be still in the process of formation. Gravitational lensing offers a theoretically less demanding alternative approach independent of the physical state and nature of the matter. In fact, the mass measurement is reliable even in merging clusters. In general, two shortcomings are recognized as affecting gravitational lensing estimates. First, projection effects, as happens also for other methods, limit the accuracy. In fact, lensing measures the mass of all the structures superimposed to the cluster (Metzler et al. 1999). Secondly, on a more theoretical ground, the steepness degeneracy makes the lensing properties invariant for a local rescaling (Saha, Read & Williams 2006). However, this can be broken having a range of source redshifts, with a very large field of view or having number counts of lensed images.

Lensing methods do not rely on equilibrium hypotheses but, even though the implicit assumption of a static mass distribution is usually made, kinematics actually affects the lensing properties of a mass distribution (Sereno 2002, and references therein). Either the peculiar motion of the deflector with respect to the Hubble flow or a coherent rotation of the matter halo brings in corrections to gravitational lensing. The nature of the two effects is substantially different (Sereno 2005a). The effect due to a translational motion comes from the local Lorentz invariance and from the existence of

★E-mail: sereno@physik.unizh.ch

the Newtonian (gravitoelectric) field (Frittelli 2003; Sereno 2005a, and references therein). On the other hand, the mass current induced by a non-null angular momentum induces a gravitomagnetic field, which is peculiar of general relativity and other metric theories of gravity, and the related dragging of inertial frames (Sereno 2002, 2003b, and references therein). The lens motion can affect observations on very different scalelengths: Galactic microlensing (Sereno 2003a), black hole lensing both in the weak (Sereno 2003a; Sereno & de Luca 2006) and the strong deflection limit (Bozza et al. 2005; Bozza, Luca & Scarpetta 2006), time-delays (Sereno 2005b) and deflection angles (Capozziello et al. 2003) in galaxy-quasar lensing can show sizable signatures of either spin or translational motion. Dark matter currents in the large-scale structure also affect the weak-lensing power spectrum even if corrections are negligible at currently accessible scales (Schäfer & Bartelmann 2006).

In this paper, I will discuss the effect of motion of galaxy clusters on gravitational lensing. The translational and rotational motions of galaxy clusters are strictly related to the formation and the evolution of large-scale structure. Peculiar velocities (Bahcall & Oh 1996; Masters et al. 2006) and spins (Bett et al. 2007; Gottloeber & Yepes 2007; Hwang & Lee 2007) can be sizable and their effect deserves attention. I will consider the peculiar lensing signatures imprinted by the motion of the cluster and how the kinematics of the deflector affects cluster mass estimators, in the weak as well as in the strong lensing regime. The paper is organized as follows. In Section 2, the properties of a model of rotating and translating lens are reviewed. Sections 3 and 4 are devoted to the effect of peculiar motions and angular momentum, respectively. Finally, Section 5 contains some concluding remarks. Throughout this paper, the reference cosmological model is, unless otherwise stated, a flat model of universe with a cosmological constant ($\Omega_{\rm M} = 0.3, \Omega_\Lambda = 0.7$) and $H_0 = 100\, h\, {\rm km\, s^{-1}\, Mpc^{-1}}$.

2 ROTATING ISOTHERMAL SPHERE

Many of the properties of galaxy clusters can be understood using a very simple model in which the matter distribution is treated as a singular isothermal sphere (SIS),

$$\rho(r) = \frac{\sigma_{\rm v}^2}{2\pi G r^2}, \tag{1}$$

where r is the radial distance, $\sigma_{\rm v}$ is the velocity dispersion and G is the gravitational constant. This model predicts quite correctly many self-similar features and scaling relations (Voit 2005). Since the total mass of a SIS is divergent, a cut-off radius much larger than the relevant length scale which characterizes the lensing phenomenon must be introduced. Based on the spherical collapse model, the limiting radius can be defined to be r_Δ, the radius within which the mean mass density is Δ times the critical density of the universe $\rho_{\rm cr} = 3H(z)/(8\pi G)$ where $H(z) = H_0\sqrt{\Omega_{\rm M}(1+z)^3 + (1-\Omega_{\rm M})}$ is the time-dependent Hubble parameter. For a SIS at redshift $z_{\rm d}$, it is (Mo, Mao & White 1998)

$$r_\Delta = \frac{2\sigma_{\rm v}}{\sqrt{\Delta} H(z_{\rm d})}. \tag{2}$$

No single definition of mass overdensity is best for all applications regarding galaxy clusters (Voit 2005). A useful approximation is based on the spherical top-hat model. For our reference ΛCDM model, $\Delta \sim 155.5$ at $z \simeq 0.3$ (Bryan & Norman 1998). Then, a halo with $\sigma_{\rm v} \sim 800\, {\rm km\, s^{-1}}$ at $z_{\rm d} = 0.3$ has a virial radius of $\sim 1.1\, {\rm Mpc}\, h^{-1}$. The total mass of a truncated SIS is

$$M_{\rm SIS} = \frac{2\sigma_{\rm v}^2}{G} r_\Delta. \tag{3}$$

The total angular momentum of a halo, J, can be expressed in terms of a dimensionless spin parameter λ, which represents the ratio between the actual angular velocity of the system and the hypothetical angular velocity that is needed to support the system (Peebles 1969; Padmanabhan 2002),

$$J \equiv \lambda \frac{G M^{5/2}}{|E|^{1/2}}, \tag{4}$$

where M and E are the total mass and the total energy of the halo, respectively. In the hypothesis of initial angular momentum acquired from tidal torquing, typical values of λ can be obtained from the relation between energy and virial radius, and the details of the spherical top-hat model (Padmanabhan 2002). The total angular momentum of a truncated SIS can be written as

$$J_{\rm SIS} = \lambda \frac{4\sigma_{\rm v}^3 r_\Delta^2}{G}. \tag{5}$$

In general, the angular velocity ω of a halo is not constant and a differential rotation should be considered (Capozziello et al. 2003). However, assuming a detailed rotation pattern does not significantly affect the results. In what follows, we will consider the case of constant angular velocity. Then, ω has to be interpreted as an effective angular velocity, $\omega \simeq J_{\rm SIS}/I_{\rm SIS}$, where $I_{\rm SIS}$ is the central momentum of inertia of a truncated SIS, $I_{\rm SIS} = (2/9) M_{\rm SIS}\, r_\Delta^2$. In terms of the spin parameter,

$$\omega = 9\lambda \frac{\sigma_{\rm v}}{r_\Delta}. \tag{6}$$

Translational or rotational motions of the deflector affect its lensing properties in very different ways (Sereno 2005a). The effect due to a translational motion is a consequence of the local Lorentz invariance applied on the standard gravitoelectric field (Frittelli 2003; Sereno 2005a). A peculiar velocity with respect to the local Hubble flow affects the lensing quantities through an overall multiplicative scaling factor. For slow motions, the factor takes the form $(1 - v_{\rm los}/c)$ where $v_{\rm los}$ is the component of the peculiar velocity along the line of sight and c is the speed of light in the vacuum (Frittelli 2003; Sereno 2005a); $v_{\rm los}$ is taken to be negative for receding lenses, that is, for peculiar motions directed far away from the observer and towards the source.

The problem of light deflection by a lens with angular momentum is very different in nature, since it is related to the dragging of inertial frames. The lensing effect of a spin depends on the details of the rotational motion (Sereno 2002). Gravitational lensing by a rotating isothermal sphere has been discussed in Sereno & Cardone (2002) and Sereno (2005b). All of the lensing properties can be derived by the projected deflection potential, ψ. For a SIS in rigid motion,

$$\psi^{\rm SIS} \simeq \left(1 - \frac{v_{\rm los}}{c}\right) x - L \left(\frac{3}{2} x_\Delta - x\right) x_1; \tag{7}$$

lengths in the lens plane $x_1 - x_2$ are in units of $R_{\rm E}$,

$$R_{\rm E} \equiv 4\pi \left(\frac{\sigma_{\rm v}}{c}\right)^2 \frac{D_{\rm d} D_{\rm ds}}{D_{\rm s}}, \tag{8}$$

where $D_{\rm d}, D_{\rm s}$ and $D_{\rm ds}$ are the angular diameter distances from the observer to the deflector, from the observer to the source and from the deflector to the source, respectively; the dimensionless virial radius is $x_\Delta = r_\Delta/R_{\rm E}$. Equation (7) holds when the angular momentum is directed along the x_2-axis. The dimensionless parameter $L \equiv (2/3)(\omega\, R_{\rm E}/c)$ is an estimate of the rotational velocity. When $L > 0$, the angular momentum of the lens is positively oriented along $\hat{x}_2$. The peculiar motion acts as a correction independent of the position in the lens plane. On the other hand, there are two main contributions to the gravitomagnetic correction (Sereno & Cardone 2002). The first contribution comes from the projected momentum of inertia inside the radius x; the second contribution is due to the mass outside x and can become significant in the case of a very extended lens, that is, for a very large cut-off radius. We remark that the global factor $(1 - v_{\rm los}/c)$ should apply overall, but in equation (7) we have neglected the higher order contribution due to its application to the gravitomagnetic term.

3 PECULIAR MOTION

The velocity field of galaxy clusters is a result of gravitational interaction of inhomogeneities in the large-scale mass distribution of the universe. The probability distribution function of cluster peculiar velocities provides a tool for distinguishing between different cosmological models with differences showing up most at the high-velocity end (Bahcall & Oh 1996). Apart from the dependence on the cosmological density parameters, velocities scale in proportion to the normalization constant of the matter power spectrum, which can be expressed in terms of σ_8, the rms mass fluctuation in a sphere of radius $8\,h^{-1}$ Mpc. This parameter must then be set by requiring that the cosmological models reproduce the observed abundance of rich clusters (Colberg et al. 1999). For a flat ΛCDM model with $\Omega_{\rm M} = 0.3, \sigma_8 = 0.90$ and $h = 0.7$, the three-dimensional velocity dispersion for clusters is $\gtrsim 340\,{\rm km\,s^{-1}}$ (Colberg et al. 1999). It is worth noting that the distribution of peculiar velocities for peaks of the smoothed initial density field, which can be conveniently associated with clusters, is independent of peak height (Colberg et al. 1999).

Peculiar velocities should decay in low-$\Omega_{\rm M}$ models (Peebles 1993). However, due to non-linear effects, the late time-growth of peculiar velocities is systematically underestimated by linear theory. Deviations are especially important for members of superclusters whose velocities are about 20–30 per cent larger than those of isolated clusters (Colberg et al. 2000).

Standard methods for determining radial peculiar velocities compare the velocity determined from the redshift with that expected for the uniform Hubble flow, $H_0\,D$, where the distance to the cluster is typically determined with an empirical relationship based on Tully–Fisher (TF) or $D - \sigma$ distance indicators. Recently, Masters et al. (2006) calibrated the TF template with a sample of 807 galaxies in the fields of 31 nearby clusters and groups. Based on a subsample of 486 bona fide cluster members, they found a cluster velocity dispersion of $298 \pm 34\,{\rm km\,s^{-1}}$, in remarkable agreement with theoretical expectations. The largest peculiar velocities were found to exceed $600\,{\rm km\,s^{-1}}$. Similar results were also obtained by the POTENT program aimed to recover the three-dimensional velocity field using the expected irrotationality of gravitational flows in the weakly non-linear regime (Bertschinger & Dekel 1989; Dekel et al. 1999).

The bulk peculiar velocity of the cluster gas can also be measured through the kinematic component of the Sunyaev–Zeldovich effect (SZE), that is, the change in the cosmic microwave background (CMB) intensity caused by scattering (Sunyaev & Zeldovich 1980; Rephaeli 1995; Holzapfel et al. 1997). This kinematic effect appears as an increment or a decrement in the CMB intensity at all frequencies. Unfortunately, actual observational uncertainties are too large to allow reliable estimates, and only limits to the bulk flow of the intermediate-redshift universe in the direction of the CMB dipole can be obtained (Benson et al. 2003).

Given the overall scaling induced by peculiar velocities on all lensing observables, the relative error in the mass estimate made when the motion along the line of sight is neglected is

$$\frac{\Delta M}{M} \simeq -\frac{v_{\rm los}}{c}. \tag{9}$$

Observations and theoretical predictions on the velocity field discussed above suggest that the systematic error is as large as $\sim 0.3 - 0.4$ per cent. Assuming a Gaussian velocity distribution, the effect is $\gtrsim 0.1$ per cent in nearly one-third of the systems.

Whereas corrections on a single estimate at the level of the per cent cannot affect in a significant way complete statistical analyses, one might wonder if very deep observations of galaxy clusters could allow a detection of the gravitational lensing kinematic effect. Since the translational motion acts as an overall multiplicative factor, there is a full degeneracy between the effect of the peculiar velocity and a re-scaling of the central mass density of the cluster. Then, even using fiducial gravitational lensing data, we cannot disentangle such a degeneracy.

A possible way to study the kinematic translational effect could be through joint analyses with independent data sets. Cross-correlations of SZE surveys with lensing data should amplify the effect (Schäfer & Bartelmann 2006). Future all-sky submillimetric telescopes, such as the *Planck* satellite,[1] will measure the thermal SZE in many thousands of galaxy clusters. However, the smaller kinetic SZE should be detected in just a few dozens. Then, for contemporary and near-future lensing surveys, the kinetic correction is not supposed to play a significant role. This is also the case considering the weak-lensing power spectrum (Schäfer & Bartelmann 2006).

4 ROTATING CLUSTERS

Angular momentum should be presumably acquired by haloes (dark matter plus gas) through tidal interactions with neighbouring objects (Peebles 1969; Doroshkevich 1970; White 1984; Bullock et al. 2001). Tidal forces are stronger in dense environments, leading to more coherent rotation. Here, we are interested in coherent rotation, whereas shear flows which imply higher order gravitomagnetic effects are not considered. Recent large N-body simulations have given a detailed picture of the spin distribution of massive haloes (Bett et al. 2007; Gottloeber & Yepes 2007; Macciò et al. 2007). The trend of the spin with the halo mass is very weak and shows a large dispersion but more-massive haloes seem to have a slightly less coherent rotation in the median. The spin for massive clusters is nearly independent of the halo shape. The distribution of spins, as obtained from independent groups, can be approximated either by a log normal distribution (Vitvitska et al. 2002; Gottloeber & Yepes 2007) or by a function with a longer tail at low λ (Bett et al. 2007), but anyway the main features of the distributions are pretty similar with a median value of $\lambda_{\rm med} \sim 0.03$ and a width of $\sigma_{\rm lg} \sim 0.2$ (Bett et al. 2007). The number of clusters with $\lambda \gtrsim 0.1$ is $\sim$2 per cent.

Direct observations of rotating galaxy clusters are much more uncertain. From a survey-level substructure analysis of 25 low richness clusters of galaxies contained in the 2dF Galaxy Redshift Survey (2dFGRS) cluster catalogue, Burgett et al. (2004) found that three clusters exhibit velocity-position characteristics consistent with the presence of possible rotation, shear or infall dynamics. Recently, Hwang & Lee (2007) searched for rotating clusters in Sloan Digital Sky Survey (SDSS) and 2dFGRS. Out of a sample of 56 galaxy clusters with enough galaxy members with known radial velocity,

[1] http://www.rssd.esa.int/index.php?project=Planck.

they selected six likely rotating ones. The estimated rotation amplitudes are in the range $190\,\mathrm{km\,s^{-1}} \lesssim v_{\rm rot} \lesssim 450\,\mathrm{km\,s^{-1}}$ whereas the tentative velocity gradients are in the range $400\,\mathrm{km\,s^{-1}\,Mpc^{-1}} \lesssim \mathrm{d}v/\mathrm{d}R \lesssim 800\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Even if the sample of clusters is not statistically complete, more than 10 per cent of the analysed clusters show signatures of a rotation pattern. The ranges in velocity extend to higher values when other six less likely rotating clusters are included in the subsample. Maybe the best case for a rotating cluster is Abell 2107, with an estimated angular velocity for the entire cluster of $\mathrm{d}v/\mathrm{d}R \sim 718\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (Kalinkov et al. 2005 and references therein).

Evidence of cluster rotation from X-ray analyses of the intracluster medium is less conclusive. In principle, the presence of gas bulk velocities can be detected through Doppler shifts of X-ray spectral lines. So far, *ASCA* (Dupke & Bregman 2005) or *Chandra* (Dupke & Bregman 2006) observations have shown evidence for velocity gradients consistent with transitory and/or rotational bulk motion in a very few clusters. Interpreting the velocity difference for regions opposed to the centre as due to residual gas circulation, Dupke & Bregman (2006) estimated a corresponding circular velocity of $\sim(1.2 \pm 0.7) \times 10^3\,\mathrm{km\,s^{-1}}$ in the Centaurus cluster. It is worth noting that some recent numerical simulations have shown that the gas spin is $\sim$1.4 times larger than the spin of dark matter with a tendency to decrease with halo mass (Gottloeber & Yepes 2007).

The angular velocity of a cluster can be expressed in term of the spin parameter and the overdensity as

$$\omega = \frac{9}{2}\lambda H(z)\sqrt{\Delta} \tag{10}$$

$$\simeq 260\left(\frac{\lambda}{0.04}\right) h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}, \tag{11}$$

where in equation (11), I have substituted for some reference values, that is, a galaxy cluster at $z_{\rm d} \simeq 0.3$ with a virial overdensity of $\Delta \simeq 155.5$. For average values of the spin, the angular velocities predicted in equation (11) are smaller than the measurements discussed above. This can be explained if rotation is more likely detected in clusters with large spin ($\lambda \sim 0.1$). The dimensionless parameter L can be written as

$$L = 3\lambda\frac{R_{\rm E}}{c/H(z)}\sqrt{\Delta} \tag{12}$$

$$\simeq 2.4 \times 10^{-5}\left(\frac{\lambda}{0.04}\right)\left(\frac{\sigma_{\rm v}}{800\,\mathrm{km\,s^{-1}}}\right)^2, \tag{13}$$

where in the second line, I have considered a galaxy cluster at $z_{\rm d} \simeq 0.3$ with a virial overdensity of $\Delta \simeq 155.5$ and a background source population at $z_{\rm s} \sim 1.5$. Since spin effects are proportional to L, we expect them to be small.

4.1 Strong lensing

Detection of giant luminous arcs in the inner regions of galaxy clusters provides a tool for one of the most direct and reliable mass estimate of the inner regions. If the cluster is not far from spherical symmetry, then at first order,

$$M(<\theta_{\rm arc}) \simeq \Sigma_{\rm cr}\pi(D_{\rm d}\theta_{\rm arc})^2, \tag{14}$$

where $\theta_{\rm arc}$ is the angular radius of the arc and the mean density inside the Einstein radius equals to the critical surface density $\Sigma_{\rm cr} = c^2\,D_{\rm s}/(4\pi\,G D_{\rm d}\,D_{\rm ds})$. Due to lens spin, the critical curve is slightly shifted by $\Delta\theta/\theta_{\rm arc} \sim L$ (Sereno 2005b). Then ignoring spin contribution affects the mass estimates by

$$\frac{\Delta M}{M} \simeq 2L. \tag{15}$$

The effect is small even for very massive and highly spinning clusters. As can be seen from equation (13), the relative error is $\lesssim$0.005 per cent for typical values of $\sigma_{\rm v} \sim 800\,\mathrm{km\,s^{-1}}$ and $\lambda \sim 0.04$, and it can be as large as 0.04 per cent for $\sigma_{\rm v} \sim 1500\,\mathrm{km\,s^{-1}}$ and $\lambda \sim 0.1$.

Differently from the translational motion, the rotation can imprint peculiar lensing signatures which allow, in principle, to distinguish the gravitomagnetic effect from that of the other mass perturbations, such as a quadrupole moment (Sereno 2005b). Despite the relative variation in lensing quantities being small, the absolute variation due to the spin can be of interest. Giant luminous arcs usually form at a radial distance of $\sim$30 arcsec. Even a very tiny relative deviation of $\lesssim$0.01 per cent brings about a correction to the deflection angle of $\sim$3 mas, at the level of the astrometric resolution obtained with ground-based optical interferometry. This could be interesting but the real observational shortcoming is due to intrinsic size of the lensed source. In fact, either the width of thin arcs or the size of images in multiple systems is larger than the astronometric shift due to the kinetic effect.

4.2 Weak lensing

In the outer regions of galaxy clusters, the deflection is small and the shear, that is, the anisotropic distortion field, produces a weak alignment of background images, which can be detected by averaging over many near images (Bartelmann & Schneider 2001). For an axially symmetric mass distribution, images are tangentially oriented relative to the direction towards the mass centre. Rotation affects the shear components. The tangential shear corresponding to the potential in equation (7) is

$$\gamma_{\rm t} \simeq \frac{1}{2x}(1 - Lx\sin\varphi) \tag{16}$$

with φ the polar angle in the lens plane. Then, the angular momentum of the lens gives rise to a modulation in the tangential shear which varies as the sine of the polar angle. For a rigid rotation, the amplitude of the signal ($\sim L/2$) is constant with the radius.

The relative systematic error made neglecting the rotation is $\lesssim L\,x$. In terms of the spin parameter, the uncertainty on the mass can be written as

$$\frac{\Delta M}{M} \lesssim 6\lambda\left(\frac{\sigma_{\rm v}}{c}\right) f_{r_\Delta} \tag{17}$$

$$\simeq 6 \times 10^{-4}\left(\frac{\lambda}{0.04}\right)\left(\frac{\sigma_{\rm v}}{800\,\mathrm{km\,s^{-1}}}\right) f_{r_\Delta}, \tag{18}$$

where $f_{r_\Delta} (=\langle r\rangle/r_\Delta)$ is the mean radius of the observed region in units of the viral radius. The field of view surrounding a massive cluster ($\sigma_{\rm v} \sim 1500\,\mathrm{km\,s^{-1}}$) can be explored up to large radii ($\lesssim 2\,\mathrm{Mpc}\,h^{-1}$). Then, for high spins ($\lambda \sim 0.1$), the corresponding error on the mass estimate is of the order of $\sim$0.3 per cent.

In principle, the typical angular modulation induced by the gravitomagnetic field provides a way to measure the angular momentum in galaxy clusters. A similar effect might be artificially detected in a static mass configuration if by mistake the assumed position of the geometrical centre of the theoretical mass model does not coincide with the actual centre of the mass distribution (Sereno 2002). However, the barycentre of a well-relaxed galaxy cluster can be easily identified with several reliable pointers, such as the location of the central brightest galaxy and the peak in the X-ray emission.

In order to assess the detectability of the effect in the weak-lensing regime, the gravitomagnetic correction must be compared to the main source of statistical uncertainty, which is due to the intrinsic ellipticity of the source galaxies, $\Delta\gamma_t/\gamma_t \simeq \sigma_e/(\sqrt{2N}\gamma_t)$, where $\sigma_e(\sim 0.2-0.3)$ is the intrinsic dispersion in background galaxy ellipticity per mode and N is the number of background sources. Uncertainties on the tangential shear $\lesssim 0.01-0.02$ are routinely obtained with ground-based observations by averaging the signal over circular annuli; the total number of annuli is usually a dozen. On the other hand, for massive clusters ($\sigma_v \simeq 1500\,\mathrm{km\,s^{-1}}$) with high spins ($\lambda \simeq 0.1$), the modulation amplitude is $\sim 10^{-4}$, two orders of the magnitude smaller than the noise.

Let us give a closer look at the effect. A coherent rotation imprints a coherent angular pattern in the lensing signal. For a nearly constant angular velocity, the signature is constant with the radius, see equation (16), which further helps in attempting to detect the signal. Then, the gravitomagnetic correction, when considered in subsequent circular annuli with increasing mean radius, can be viewed as a periodic function of the polar angle with period 2π. The detection of a modulation is much easier to extract than a steady signal. Since the modulation is a sine function with a minus sign, the tangential shear is enhanced in the southern part of the cluster, that is, $\pi < \varphi < 2\pi$, and vice versa in the north. Let us consider the tangential distortion in the four quadrants of a circular annulus. The average tangential shear signal is $1/(x_{\max} + x_{\min})$, where $x_{\max}$ and $x_{\min}$ are the outer and the inner radius of the annulus, respectively. In the north, that is, first and second quadrant, the average signal is suppressed by $-L/\pi$; in the south, that is, third and fourth quadrant, the signal is enhanced by $+L/\pi$. If the shear signal is averaged over the whole annulus, the gravitomagnetic contribution is washed out for a circular mass distribution. Whenever the total amplitude variation of the gravitomagnetic signal ($\sim L$) is larger than the statistical error due to the intrinsic ellipticity, there is a clear detection of the gravitational effect of the rotation. Unfortunately, this condition is fulfilled only for surface densities of the background galaxy sources, ρ_{back}, well beyond actual technological capabilities. Considering a massive cluster with a large spin whose weak-lensing signal is collected over large circular sectors with inner radius of $\sim 2R_{\mathrm{E}}$ (to excise the central strong lensing region) and extending up to the viral radius r_Δ, the gravitomagnetic tangential shear can be detected only if $\rho_{\mathrm{back}} \gtrsim 10^3$ galaxies per arcmin^2.

Future space-borne missions or the next-generation ground-based telescopes should substantially increase the observed densities of background galaxies with respect to the actual values, but not enough. As an example, the proposed SNAP mission[2] should get $\rho_{\mathrm{back}} \sim 10^2$ galaxies per arcmin^2, well below the requirements for the gravitomagnetic detection.

5 CONCLUSIONS

Kinematics affects mass measurements based on gravitational lensing. In order to give a quantitative estimate, I have considered as lens model a SIS in rigid rotation and in translational motion with respect to the background. In fact, increasing the accuracy either by considering a rotational velocity dependent on radius or by a mass density profile predicted by numerical simulations would not affect results in a sensible way. Peculiar motions or coherent rotations act very differently as regards gravitational lensing but systematic deviations turn out to be $\lesssim 1$ per cent, well below actual statistical uncertainty or projection effects. The kinematic effect should not have a sizable impact on present and near-future statistical studies on scaling relations in galaxy clusters.

As regards the detectability of the kinematic effect in galaxy clusters in the near future, prospects are not so good. The effect of translational motion can be sizable but is degenerate with an overall mass-rescaling: gravitational lensing observations by their own cannot disentangle the effect. On the other hand, angular momentum imprints a distinctive feature. Due to the axially symmetric intrinsic gravitomagnetic field induced by rotation, the tangential shear shows a angular amplitude modulation and a consequent north–south asymmetry. Unfortunately, the effect is very tiny and even very deep exposures lack the required (very high) background source density.

[2] http://snap.lbl.gov/.

ACKNOWLEDGMENTS

This work benefited from the careful reading and the argued criticism of the referee. The author is supported by the Swiss National Science Foundation and by the Tomalla Foundation.

REFERENCES

Bahcall N. A., Oh S. P., 1996, ApJ, 462, L49
Bartelmann M., Schneider P., 2001, Phys. Rep., 340, 291
Benson B. A. et al., 2003, ApJ, 592, 674
Bertschinger E., Dekel A., 1989, ApJ, 336, L5
Bett P., Eke V., Frenk C. S., Jenkins A., Helly J., Navarro J., 2007, MNRAS, 376, 215
Bozza V., de Luca F., Scarpetta G., Sereno M., 2005, Phys. Rev. D, 72, 083003
Bozza V., de Luca F., Scarpetta G., 2006, Phys. Rev. D, 74, 063001
Bryan G. L., Norman M. L., 1998, ApJ, 495, 80
Bullock J. S., Dekel A., Kolatt T. S., Kravtsov A. V., Klypin A. A., Porciani C., Primack J. R., 2001, ApJ, 555, 240
Burgett W. S. et al., 2004, MNRAS, 352, 605
Capozziello S., Cardone V. F., Re V., Sereno M., 2003, MNRAS, 343, 360
Colberg J. M., White S. D. M., MacFarland T. J., Jenkins A., Pearce F. R., Frenk C. S., Thomas P. A., Couchman H. M. P., 2000, MNRAS, 313, 229
Dekel A., Eldar A., Kolatt T., Yahil A., Willick J. A., Faber S. M., Courteau S., Burstein D., 1999, ApJ, 522, 1
Doroshkevich A. G., 1970, Astrophysics, 6, 320
Dupke R. A., Bregman J. N., 2005, ApJS, 161, 224
Dupke R. A., Bregman J. N., 2006, ApJ, 639, 781
Frittelli S., 2003, MNRAS, 340, 457
Gottloeber S., Yepes G., 2007, ApJ, 664, 117
Holzapfel W. L., Ade P. A. R., Church S. E., Mauskopf P. D., Rephaeli Y., Wilbanks T. M., Lange A. E., 1997, ApJ, 481, 35
Hwang H. S., Lee M. G., 2007, ApJ, 662, 236
Kalinkov M., Valchanov T., Valtchanov I., Kuneva I., Dissanska M., 2005, MNRAS, 359, 1491
Macciò A. V., Dutton A. A., van den Bosch F. C., Moore B., Potter D., Stadel J., 2007, MNRAS, 378, 55
Masters K. L., Springob C. M., Haynes M. P., Giovanelli R., 2006, ApJ, 653, 861
Metzler C. A., White M., Norman M., Loken C., 1999, ApJ, 520, L9
Mo H. J., Mao S., White S. D. M., 1998, MNRAS, 295, 319
Nagai D., Vikhlinin A., Kravtsov A. V., 2007, ApJ, 655, 98
Padmanabhan T., 2002, Theoretical Astrophysics, Volume III: Galaxies and Cosmology. Cambridge Univ. Press, Cambridge, UK
Peebles P. J. E., 1969, ApJ, 155, 393

Peebles P. J. E., 1993, Principles of Physical Cosmology. Princeton Univ. Press, Princeton, NJ
Rasia E. et al., 2006, MNRAS, 369, 2013
Rephaeli Y., 1995, ARA&A, 33, 541
Saha P., Read J. I., Williams L. L. R., 2006, ApJ, 652, L5
Schäfer B. M., Bartelmann M., 2006, MNRAS, 369, 425
Sereno M., 2002, Phys. Lett. A, 305, 7
Sereno M., 2003a, MNRAS, 344, 942
Sereno M., 2003b, Phys. Rev. D, 67, 064007
Sereno M., 2005a, MNRAS, 359, L19
Sereno M., 2005b, MNRAS, 357, 1205
Sereno M., Cardone V. F., 2002, A&A, 396, 393
Sereno M., de Luca F., 2006, Phys. Rev. D, 74, 123009
Sunyaev, R. A., Zeldovich, I. B., 1980, MNRAS, 190, 413
Vitvitska M., Klypin A. A., Kravtsov A. V., Wechsler R. H., Primack J. R., Bullock J. S., 2002, ApJ, 581, 799
Voit G. M., 2005, Rev. Mod. Phys., 77, 207
White S. D. M., 1984, ApJ, 286, 38

This paper has been typeset from a TEX/LATEX file prepared by the author.